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CARTOGRAPHIC DATA ACQUISITION AND SUPPORTING ACTIVITIES: CONVENTIONAL AND SATELLITE GEODESY

World geodetic system 1984: A modern and accurate global reference frame

Paper submitted by the Defense Mapping Agency**

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The Defense Mapping Agency has developed the World Geodetic System (WGS) 1964; it is a state-of-the-art global geodetic reference system based on the use of data, techniques, and technology available within DMA through early 1984 and replaces its predecessor WGS 72. The WGS 84 reference frame, Earth Gravitational Model, geoid, and datum transformation parameters (with the local datums) are more accurate and relate more datums (83 compared to 27 for WGS 72).

These improvements can be translated into more accurate maps and charts, geodetic positioning, geoidal heights, improved satellite orbits, and the capability to relate many more local datums worldwide to a unifieid system.

Introduction

In this complex world where numerous Mapping, Charting, Geodetic , and digital products are defined in various local and/or regional geodetic datums, it becomes a straight forward requirement to simplify MC&G complexity by referencing all the products to a common reference frame globally. With this need in mind, the Defense Mapping Agency (DMA) has been actively involved since 1960 in the development of World Geodetic System (WGS). To date, four such systems, viz., WGS 60, WGS 66, WGS 72, and WGS 84, each successively more accurate, have been developed.

The latest WGS 84 represents DMA's state-of-the-art modeling of the earth from a geometric, geodetic, and gravitational standpoint using data, techniques, and technology available through early 1984.

The Reference Frame

The origin of the WGS 84 reference frame is the earth's center of mass and the Z- and X-axes are identical to the Conventional Terrestrial System as defined by the Bureau International de l'Heure for the epoch 1984.0 (Figure 1). This frame constitutes a mean or standard earth rotating at a constant rate (ω) around an average astronomic pole fixed in time.

In turn, the WGS 84 reference frame is related to an Instantaneous Terrestrial System and a Conventional Inertial System:

$$\text{WGS 84 (CTS)} = [A]\ [B]\ [C]\ [D]\ \text{CIS} \qquad (1)$$

Origin - Earth's center of mass

Z-Axis = Parallel to the direction of the Conventional Terrestrial Pole for polar motion, as defined by the Bureau I International de L'Heure on the basis of the coordinates adopted for the Bureau International de L'Heure stations.

X-Axis = Intersection of the WGS 84 Reference Meridian Plane and the plane of the Conventional Terrestrial Pole's Equator, the Reference Meridian being parallel to the Zero Meridian defined by the Bureau International de L'Heure on the basis of the coordinates adopted for the Bureau International de L'Heure stations.

Y-Axis = Completes a right-handed, earth centered, earth fixed orthogonal coordinate system, measured in the plane of the Conventional Terrestrial Pole Equator, 90° East of the X-Axis.

BIH-Defined Conventional Terrestrial Pole (1984.0)

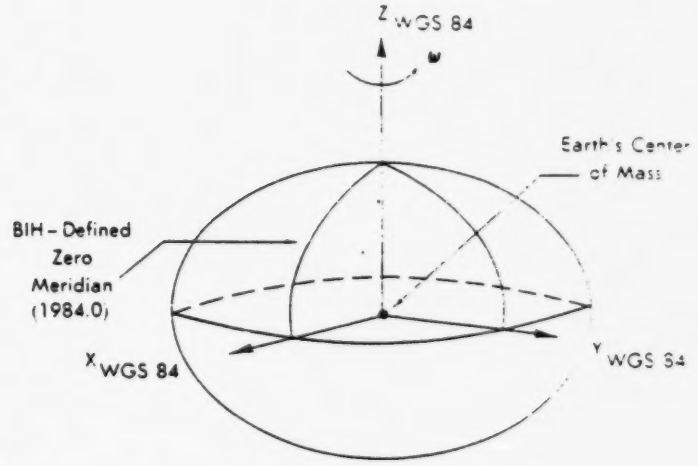

Figure 1. World Geodetic System 1984 Reference Frame.

where the rotation matrices for polar motion [A] sidereal time [B], nutation LC], and precession [D] are from the FK 5 System referenced to Epoch J2000.0.

For practical realization, the WGS 84 reference frame or coordinate system was defined by suitably modifying the NSWC 9Z-2 coordinate system of the Navy Navigation Satellite System. This modification consisted of the removal of biases in the origin, scale, and longitude definition of the Doppler system and is defined as:

$$Z(\text{WGS 84}) = Z(\text{NSWC 9Z-2}) + 4.5 \text{ meters} \quad (2)$$

$$S(\text{WGS 84}) = S(\text{NSWC 9Z-2}) - 0.6 \text{ ppm} \quad (3)$$

$$\lambda(\text{WGS 84}) = \lambda(\text{NSWC 9Z-2}) + 0.814'' \quad (4)$$

In the above relationships, Equations 2-4 refer to the Z-axis bias, scale correction, and longitudinal bias in the definition of the prime meridian, respectively, and the WGS 84, thus achieved, is coincident with the Bureau International de L'Heure-defined CTS[1].

The Defining Parameters and Associated Constants

In geodetic considerations, three different surfaces or earth figures are normally involved and used. In addition to the earth's actual physical surface, the other two include a geometric (or mathematical) reference surface, the ellipsoid, and an equipotential surface, the geoid.

In determining the WGS 84 Ellipsoid and its associated defining parameters, the WGS 84 Development Committee decided to adopt the International Union of Geodesy and Geophysics defined Geodetic Reference System 1980 as its reference.

The WGS 84 Ellipsoid, as an integral component of the system for the earth's geometric figure and theoretical gravity definition, is a geocentric, equipotential, ellipsoid of revolution; Table 1 lists the four defining parameters adopted from the Geodetic Reference System 80[2], except for one minor exception. The WGS 84 defines C_2 instead of J_2 of Geodetic Reference System 80.

Other associated constants adopted and used in WGS 84 are given in Table 2.

Table 1
WGS 84 Ellipsoid
Four Defining Parameters

Parameters	Notation	Magnitude	Accuracy (1 σ)
Semimajor Axis	a	6378137 m	±2 m
Normalized Second Degree Zonal Harmonic Coefficient of the Gravitational Potential	$\bar{C}_{2,0}$	$-484.16685 \times 10^{-6}$	$\pm 1.30 \times 10^{-9}$
Angular Velocity of the Earth	ω	7292115×10^{-11} rad s^{-1}	$\pm 0.1500 \times 10^{-11}$ rad s^{-1}
The Earth's Gravitational Constant (Mass of Earth's Atmosphere Included)	GM	3986005×10^{8} m^3 s^{-2}	$\pm 0.6 \times 10^{8}$ m^3 s^{-2}
Parameter Values for Special Applications			
The Earth's Gravitational Constant (Mass of Earth's Atmosphere Not Included)	GM′	3986001.5×10^{8} m^3 s^{-2}	$\pm 0.6 \times 10^{8}$ m^3 s^{-2}
Angular Velocity of the Earth (In a Precessing Reference Frame)	ω*	$(7292115.8553 \times 10^{-11} + 4.3 \times 10^{-15} T_U)$ rad s^{-1}	$\pm 0.1500 \times 10^{-11}$ rad s^{-1}

T_U = Julian Centuries From Epoch J2000.0

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World Geodetic System 1984

Table 2
Relevant Miscellaneous Constants and Conversion Factors

Constant	Symbol	Numerical Value
Velocity of Light (in a Vacuum)	c	299792458 m s^{-1}
Dynamical Ellipticity	H	1/305.4413
Earth's Angular Velocity [for Satellite Applications; see Equation (3-14), Reference 3.]	ω^*	$(7292115.8553 \times 10^{-11} + 4.3 \times 10^{-15} T_U)$ rad s^{-1}
Universal Constant of Gravitation	G	$6.673 \times 10^{-11} m^3 s^{-2} kg^{-1}$
GM of the Earth's Atmosphere	GM_A	$3.5 \times 10^8 m^3 s^{-2}$
Earth's Gravitational Constant (Excluding the Mass of the Earth's Atmosphere)	GM'	$3986001.5 \times 10^8 m^3 s^{-2}$
Earth's Principal Moments of Inertia (Dynamic Solution)	A	8.0091029×10^{37} kg m^2
	B	8.0092559×10^{37} kg m^2
	C	8.0354872×10^{37} kg m^2

Conversion Factors

1 Meter	= 3.28083333333 US Survey Feet
1 Meter	= 3.28083989501 International Feet
1 International Foot	= 0.3048 Meter (Exact)
1 US Survey Foot	= 1200/3937 Meter (Exact)
1 US Survey Foot	= 0.30480060960 Meter
1 Nautical Mile	= 1852 Meters (Exact)
	= 6076.10333333 US Survey Feet
	= 6076.11548556 International Feet
1 Statute Mile	= 1609.344 Meters (Exact)
	= 5280 International Feet (Exact)

T_U = Julian Centuries from Epoch J2000.0

The Gravity Formula

In many applications, such as the computation of gravity anomalies, theoretical (or normal) gravity (γ) is required as a reference value. Values of γ_Φ in the WGS 84 (for any latitude Φ) can be computed using the closed formula:

$$\gamma_\Phi = \gamma_e \frac{(1 + k \sin^2 \Phi)}{(1 - e^2 \sin^2 \Phi)^{1/2}} \qquad (5)$$

where,

γ_e = normal gravity at the equator

= 978032.67714 mgal

k = 0.00193185138639

e^2 = 0.00669437999013.

The Earth Gravitational Model

The Earth Gravitational Model (EGM) of the WGS 84 is a spherical harmonic expansion of the earth's gravitational potential and is defined complete through degree (n) and order (m) 180, comprising 32,755 coefficients. However, only the coefficients through n = M = 18 are unclassified [3].

Accuracy values are not available for all the WGS 84 EGM coefficients; however, an error covariance matrix is available only for coefficients through n = m = 41, which were determined from the weighted least squares solution

The Geoid

In addition to the earth's geometric surface or figure, the WGS 84 geoid, as the equipotential figure of the earth (also approximated by mean sea level over the oceans), is defined as so many meters above (+ N) or below (- N) the WGS 84 ellipsoid, where "N" is known as geoidal height or undulation.

The worldwide geoidal heights were calculated using the WGS 84 EGM through n = m = 180, and they can also be depicted as a contour chart (showing deviations from the WGS 84 ellipsoid) or as a grid of desired density. Figure 2 shows a worldwide WGS 84 geoid chart developed from a worldwide 1° x 1° grid using the unclassified EGM coefficients through n = m = 18.

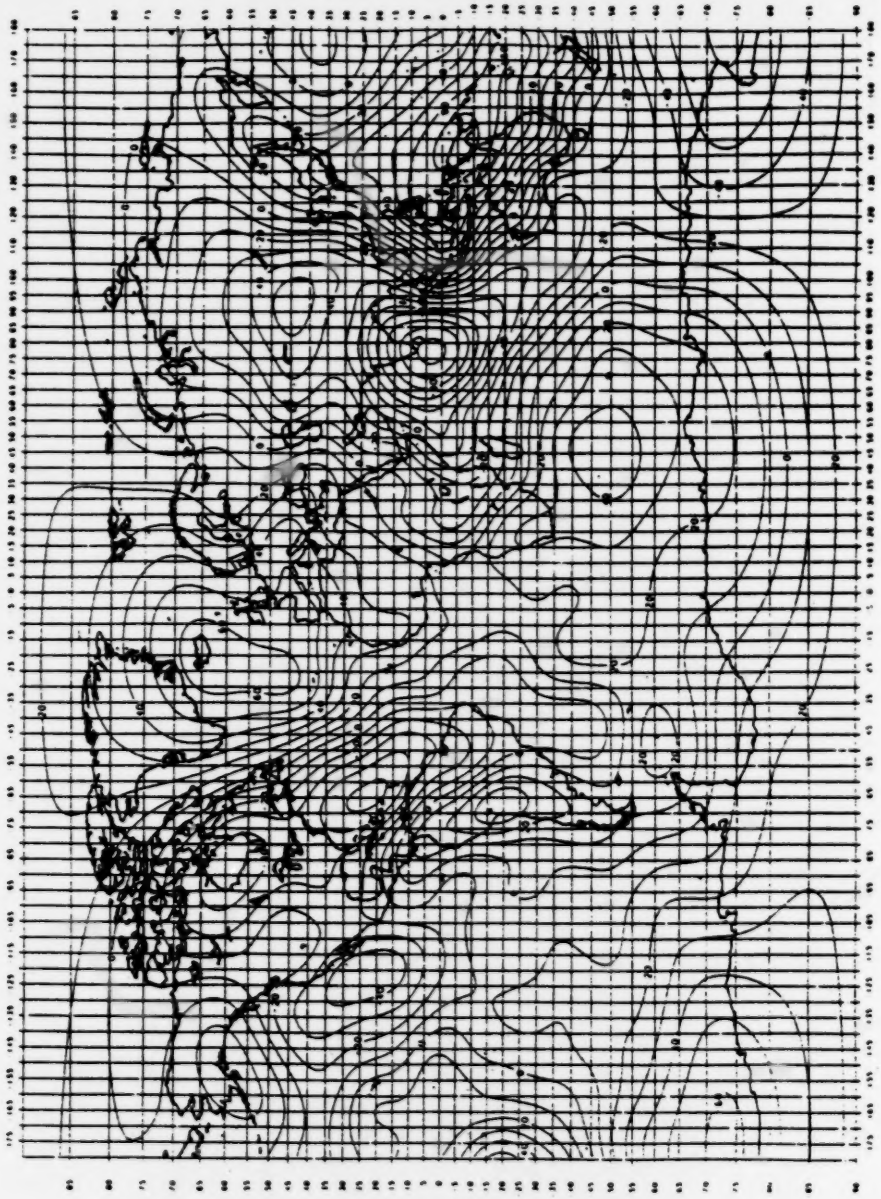

Figure 2. WGS 84 Geoid (n = m = 18 Truncation) Referenced to WGS 84 Ellipsoid ($\overline{\delta C}_{4,0}$, $\overline{\delta C}_{6,0}$, $\overline{\delta C}_{8,0}$, and $\overline{\delta C}_{10,0} \neq 0$; Units = Meters).

The Root Mean Square geoidal height for WGS 84, taken worldwide, is 30.5 meters and the error ranges from plus or minus 2 to plus or minus 6 meters (1 σ). The accuracy of the WGS 84 geoid is better than plus or minus 4 meters over approximately 93% of the globe.

Relationship with Local Geodetic Datums

Counting islands and/or other "astro" datums, the number of local geodetic datums available for mapping, charting, and geodesy requirements and applications exceeds several hundred. If the inherent technical difficulties of dealing with these numerous local datums, each defined with its own specifications and basic limitations, are considered in daily usage, the picture is just too complex and almost chaotic.

Under this bleak scenario, one of the principal purposes of a World Geodetic System is to provide the means whereby these numerous local geodetic datums can be referenced to a common system (or to each other indirectly through extrapolation) and then, WGS can facilitate simplification of the global mapping, charting, and geodesy problem.

To achieve the referencing of a local datum to WGS 84, one major requirement is to have well-distributed control points common to both the systems. DMA maintains a worldwide database of Navy Navigation Satellite System Doppler station. A search of this database produced 1591 good quality Doppler stations, which also had coordinates defined in the local datum of the area.

These 1591 Doppler stations cover 83 local geodetic datums spread out over all the six continents [3]. From a high of 405 Doppler stations common with the North American Datum 1927 in the contiguous United States, there are 29 datums with only one common station. This limitation of not having any check station thus affects about 35% (29 of 83) of the datums.

As the local geodetic datums are generally defined only horizontally and provide mean sea level (msl) heights from separately defined vertical datums the geodetic heights in the local datum (H_{LD}), required to compute datum transformation parameters, were generated using the following equation:

$$H_{LD} \approx h_{msl} + N_{LD} \qquad (6)$$

In the above equation, the local datum geoidal heights (NLD) were obtained by appropriate transformation from the WGS 84 geoidal heights. These local geoids are absolute (contrary to the relative astro-geodetic that are available for a few of the local datums) and consistent in definition with each other and also with the WGS 84 geoid worldwide.

Table 3
Transformation Parameters
—Local Geodetic Systems to WGS 84—
(For Complete Table—See Reference 4.)

Local Geodetic Systems*	Reference Ellipsoids and Parameter Differences**			Number of Doppler Stations Used to Determine Transformation Parameters	Transformation Parameters**		
	Name	Δa(m)	Δf × 10^4		ΔX(m)	ΔY(m)	ΔZ(m)
PROVISIONAL SOUTH AMERICAN 1956	International	−251	−0.14192702				
Mean Value (Bolivia, Chile, Colombia, Ecuador, Guyana, Peru, and Venezuela)				63	−288	175	−376
PUERTO RICO	Clarke 1866	−69.4	−0.37264639				
Puerto Rico and Virgin Islands				11	11	72	−101
QATAR NATIONAL	International	−251	−0.14192702				
Qatar				3	−128	−283	22
QORNOQ	International	−251	−0.14192702				
South Greenland				2	164	138	−189
REUNION	International	−251	−0.14192702				
Mascarene Island				1	94	−948	−1262
ROME 1940	International	−251	−0.14192702				
Sardinia Island				1	−225	−65	9

* Geoid heights computed using spherical harmonic expansion and WGS 84 EGM coefficient set (n = m = 180), then referenced to the ellipsoid and orientation associated with each of the local geodetic systems.

** WGS 84 minus local geodetic system.

Table 3 provides a sample listing of the transformation parameters between the 83 local datums and the WGS 84; a full listing is available elsewhere.[4]

In addition to the 83 local datums related to the WGS 84 through Doppler ties to the local control, transformation parameters (based on non-Doppler information) are also available for seven additional local datums.[4]

Accuracy

The accuracy of the WGS 84 coordinates of a site is significantly influenced by the method used to determine the coordinates. Table 4 lists the four methods generally available to establish the coordinate of a WGS 84 site and the associated accuracies achievable through each of the methods.

From Table 4, it is noticed that Method 1 (where a WGS 84 site is established through direct satellite observational data) gives the most accurate positional fix of 1-2 meters. Method 4 (the least accurate) is entirely dependent on the local/regional distortion of the local geodetic datum.

Table 4
Methods of Determining and Accuracy of WGS 84 Coordinates

Method of Determining WGS 84 Coordinates	Achievable Accuracies (1 σ)	
1. Directly Established in WGS 84 Coordinate System via a Satellite Point Positioning Solution	Φ and H	± 1 m ± 1 to 2 m
2. By Transformation from Doppler (NSWC 9Z-2) Coordinates by Bias Removal	Φ and H	± 2 m ± 2 to 3 m
3. By Transformation of WGS 72 Coordinates:		
(a) At Doppler Sites	Same as 2., Above	
(b) At non-Doppler Sites where WGS 72 Coordinates were Obtained *Indirectly* from Local Datums	Dependent on the Originating Local Datum Coordinates and Transformation Errors*	
4. By Transformation of Local Datum Coordinates	Same as 3.(b) Above	

* See DMA TR 8350.2-A, 1 December 1987.

of 1-2 meters. Method 4 (the least accurate) is entirely dependent on the local/ regional distortion of the local geodetic datum.

References

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2. Moritz, H., "Geodetic Reference System 1980," Bulletin Geodesique, 54(3):00-00, 1980.

3. Department of Defense World Geodetic System 1984, Its Definition and Relationships with Local Geodetic Systems; DMA TR 8350.2, Washington, DC, 30 September 1987.

4. Supplement to U.S. Department of Defense World Geodetic System 1984 Technical Report; Part II-Parameters, Formulas, and Graphics for the Practical Application of WGS 84; DMA TR 8350.2-B, Washington, DC, 1 December 1987.